

THE PUZZLING SIDE OF CHESS

Jeff Coakley

LOOPOLOGY I

One Piece Double Loops

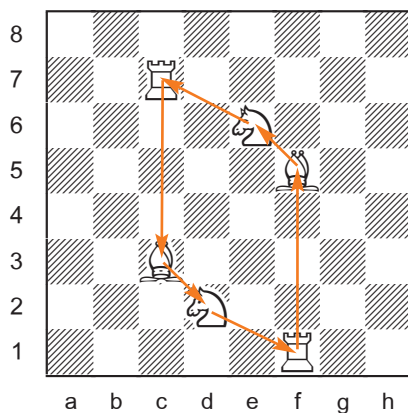
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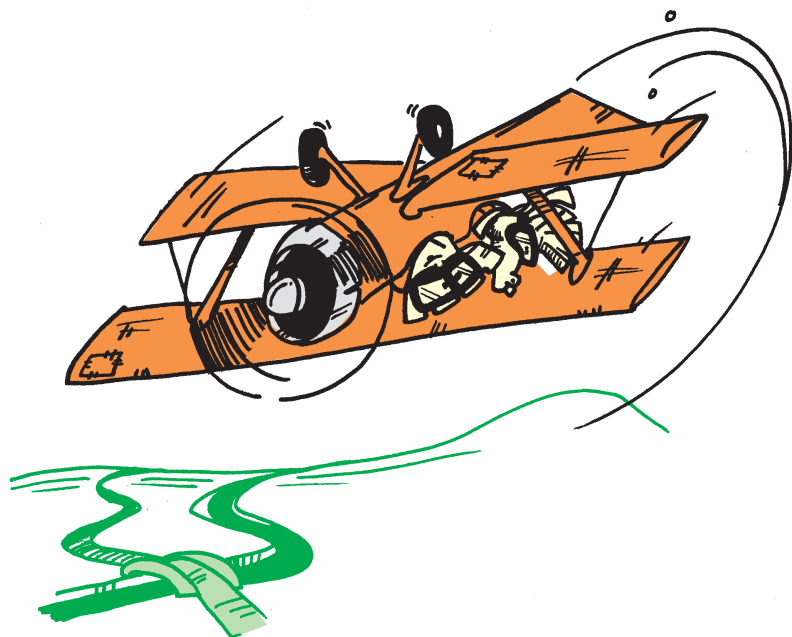
This column is the first in a series on the topic of defensive loops, a type of problem that has appeared many times on the *Puzzling Side*.

There are two basic types of *defensive loops*. One type is a “single loop” in which every piece guards exactly one piece and every piece is guarded exactly once, with the additional requirement that the chain of defence must form a continuous loop. The first piece guards the second, the second guards the third, and so on. In the end, the final piece guards the first. Though of course, there is no actual end or beginning to a loop!

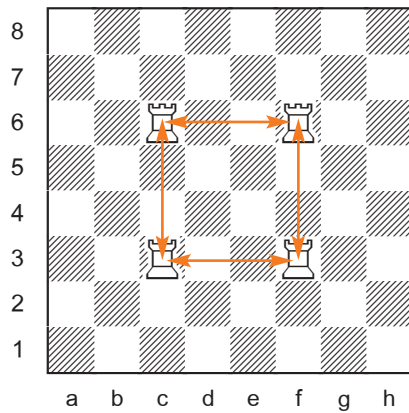
This simple example is a single loop with two rooks, two bishops, and two knights. Except for pawns, no piece in a single loop can be linked to another of the same type. Here the order is R-B-N-R-B-N.



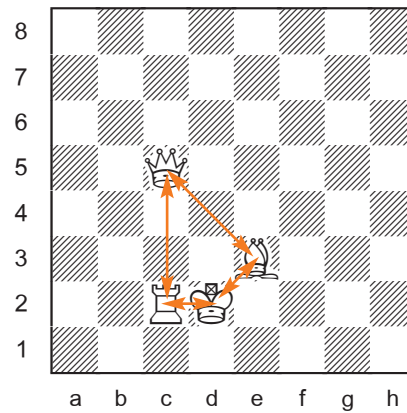
single loop



The second type of defensive loop is a “double loop” in which every piece is guarded exactly twice, by the two other pieces it is linked to in a continuous chain. The diagrams below show double loops with four rooks and with KQRB. In double loops, the linked pieces must be able to move in the same direction. This means, most notably, that a knight cannot be used to defend a line piece.



one-piece double loop



multi-piece double loop

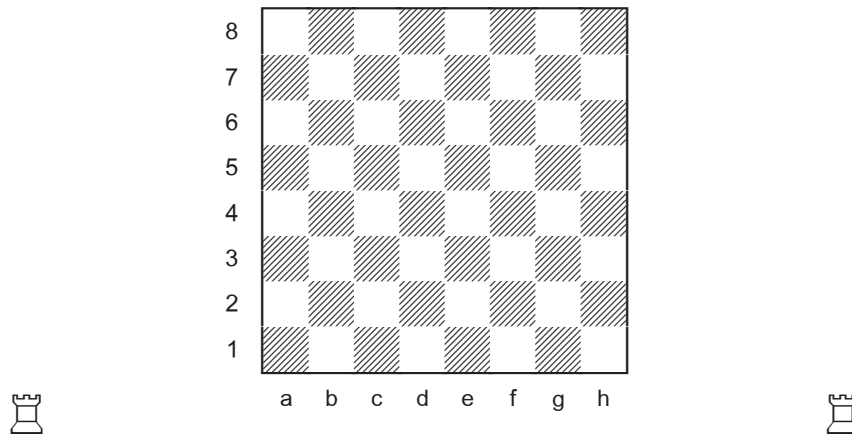
The rest of this column is devoted to double loop problems in which each piece is the same kind. Except for the *double rook loop*, all ‘one-piece double loops’ have been published previously on the *Puzzling Side*. Two of them (queens and knights) will be reprinted here because new information concerning the solutions is available.

A double loop with pawns only is impossible since they cannot reciprocally protect each other.



When Up Is Down

Double Defensive Rook Loop



Place the maximum number of rooks on the board so that each rook is defended exactly twice, by the two other rooks it is linked to in a continuous chain.

As mentioned in columns 143 and 147, François Labelle has written a program for solving defensive loops. The fruits of his extensive research will be presented this summer in the Loopology series.

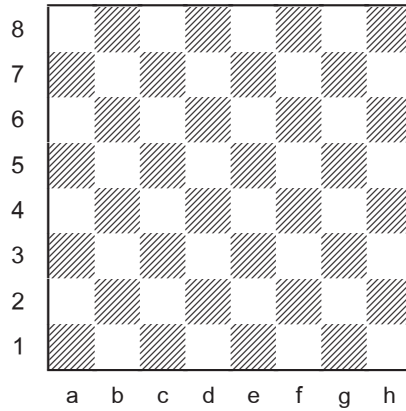
Among his many findings, he calculated that there are 168 solutions for a 12-bishop double loop (column 82) and a mere 6 for a 31-king double loop (column 151).

For a comparison of the maximum number of pieces in a double loop for pieces of different types on boards of various sizes, see the chart and text at the end of the solution section.



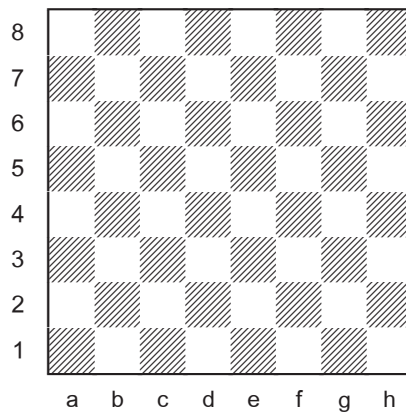
Double Looper

Double Defensive Queen Loop



Place the maximum number of queens on the board so that each queen is defended exactly twice, by the two other queens it is linked to in a continuous chain.

Double Defensive Knight Loop



Place the maximum number of knights on the board so that each knight is defended exactly twice, by the two other knights it is linked to in a continuous chain.



There and back.

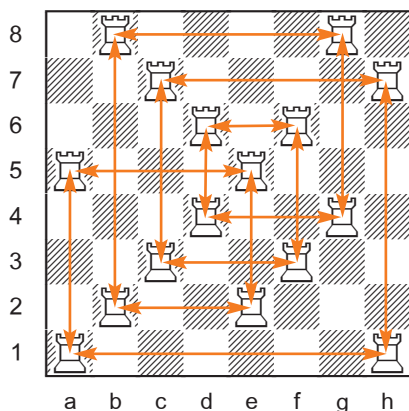
SOLUTIONS

All puzzles by J. Coakley, *Puzzling Side of Chess* (2018). The queen and knight double loops appeared previously in columns 59 and 140. Additional solutions, computer analysis for number of possible solutions, and the loops on 7 x 7 boards by François Labelle.

PDF hyperlinks. You can advance to the solution of any puzzle by clicking on the underlined title above the diagram. To return to the puzzle, click on the title above the solution diagram.

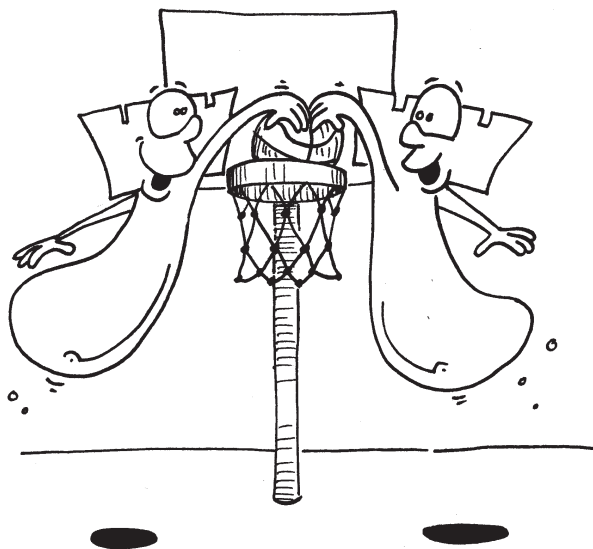
Archives. Past columns and a detailed index of problem-types and composers are available in the *Puzzling Side of Chess* archives.

Double Defensive Rook Loop



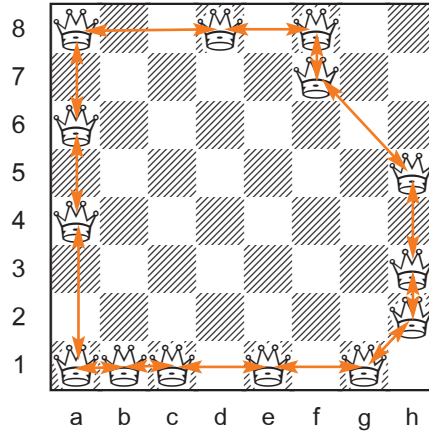
16 rooks, each defended twice in a continuous chain.

The easiest of all double loops. There are a remarkable 198,680,458 patterns for the solution. What are the chances of two people finding the same position?



Double Rook Hoops

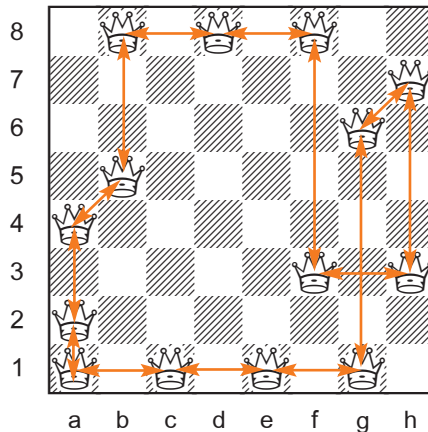
Double Defensive Queen Loop



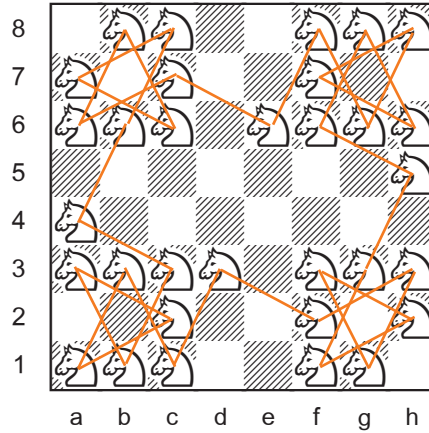
14 queens, each defended twice in a continuous chain.

The solution given above is from *Puzzling Side* 59 (January 2014). François Labelle's program has confirmed that 14 queens is the maximum and has discovered one additional solution, shown below. Two patterns is all there are. Semi-unique.

François Labelle 2018



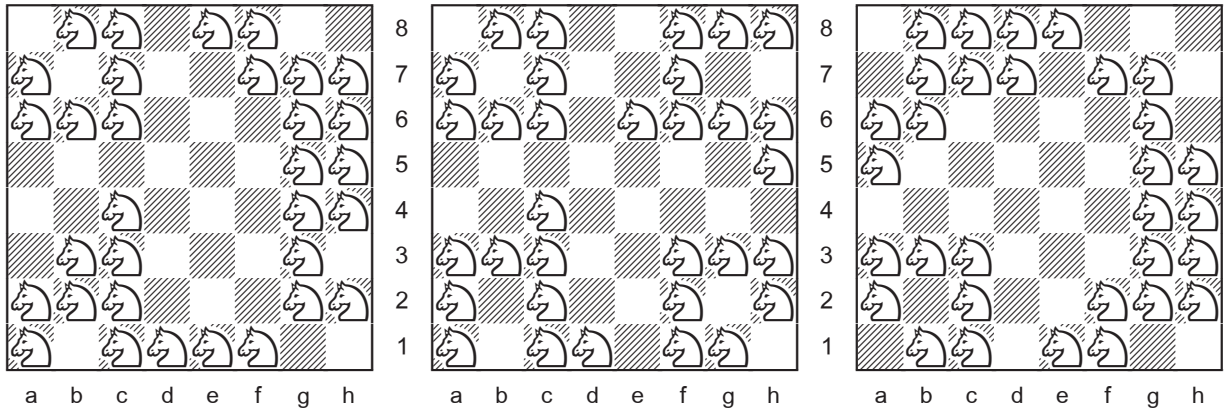
Double Defensive Knight Loop



32 knights, each defended twice in a continuous chain.

A theoretical maximum, in a symmetrical formation. This diagram is from *Puzzling Side* 140 (November 2017). The mighty engine of François Labelle, cranking the 1's and 0's in Montréal, Québec, has found three more solutions. I will leave it to you to connect the dots.

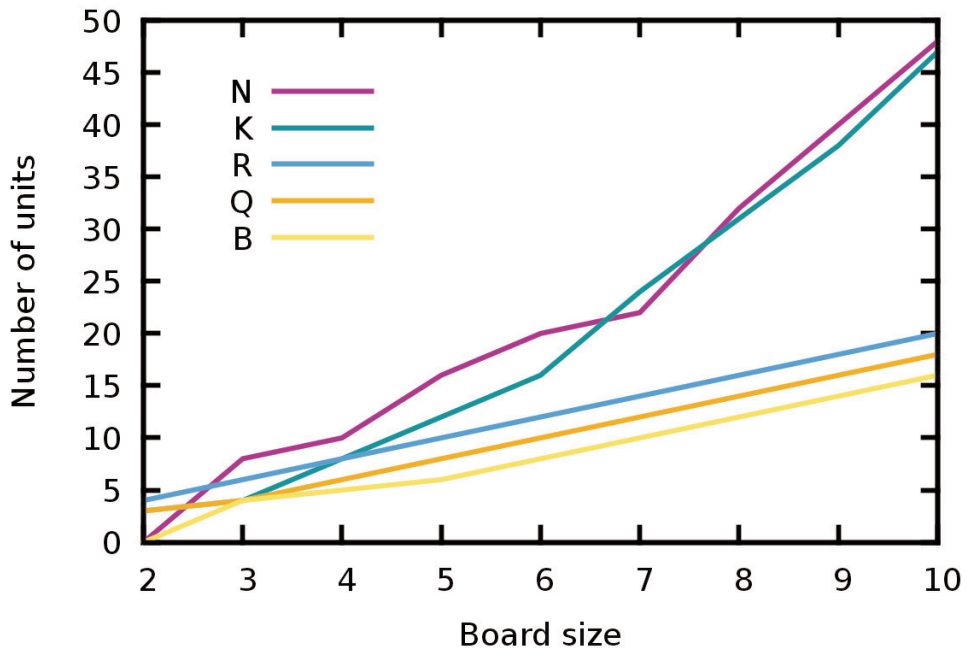
François Labelle 2018



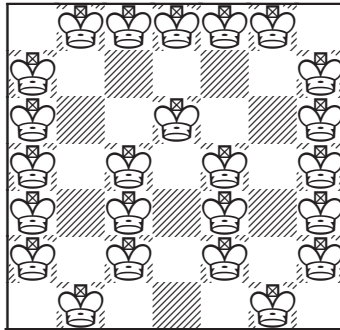
An interesting feature of his program is its ability to solve loops on boards of different sizes. The table at the right shows the maximum number of pieces of each kind in a double loop on boards 2 x 2 through 10 x 10.

See the next page for a graph and two examples on a 7 x 7 board, the only size where the maximum for a king is greater than for a knight, if we ignore the mini 2 x 2 board.

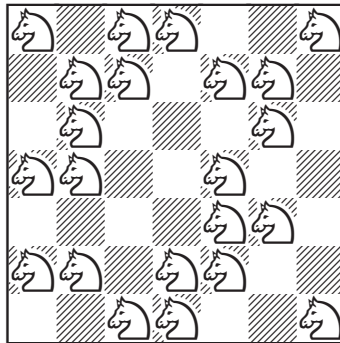
Maximum Pieces in Double Loop					
	N	B	R	Q	K
2x2	0	0	4	3	3
3x3	8	4	6	4	4
4x4	10	5	8	6	8
5x5	16	6	10	8	12
6x6	20	8	12	10	16
7x7	22	10	14	12	24
8x8	32	12	16	14	31
9x9	40	14	18	16	38
10x10	48	16	20	18	47



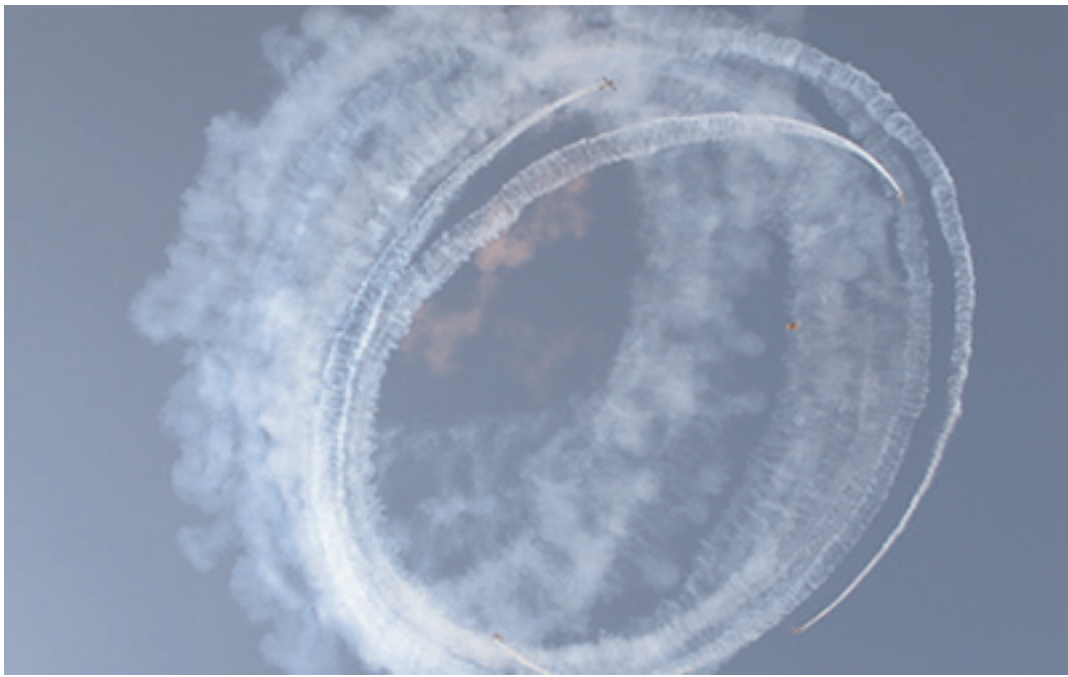
François Labelle 2018



24-king double loop on 7 x 7 board
A unique solution.



22-knight double loop on 7 x 7 board
King 24 - Knight 22
Long live the king.



Gone loopy!

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