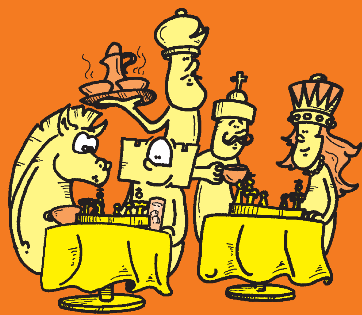


THE PUZZLING SIDE OF CHESS

Jeff Coakley

AVERAGE MOBILITY:
An Inspired Recalculation

number 147 April 21, 2018

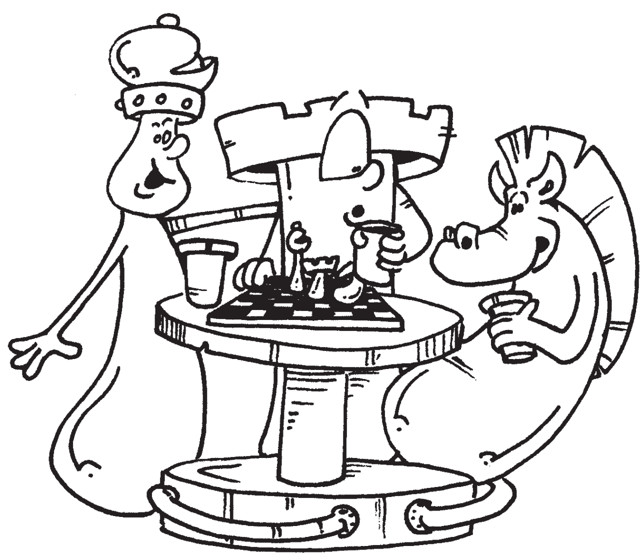
This edition of the *Puzzling Side* returns to a topic from column 24, in which the average mobility of the pieces on an empty board was calculated. The values were the following.

R = 14 moves
B = 8.75 moves
N = 5.25 moves
Q = 22.75 moves

Three surprising conclusions were drawn from these numbers.

R = B + N
Q + N = (2 x R)
Q - N = (2 x B)

Not to worry. There are some “real puzzles” to solve later.

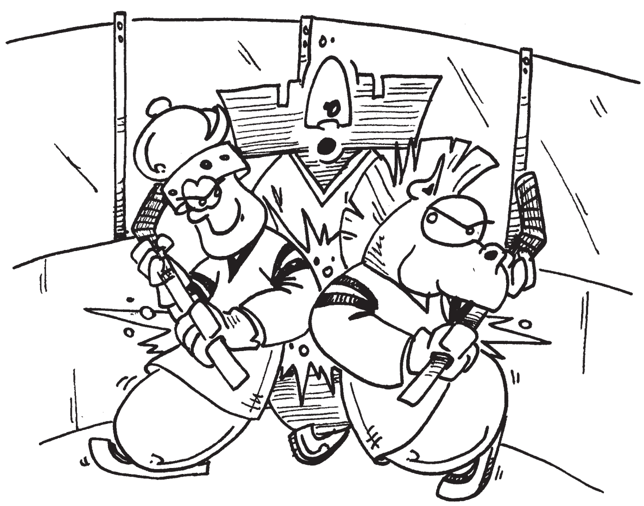


The Value of Good Friends

Earlier this year, while discussing *defensive loop* problems, I asked François Labelle what he thought of these calculations. As many of you already know, François is a chess composer from Montreal, widely recognized for his expertise in computer programming.

He noted that the averages on an empty board were the upper limits of mobility and proposed taking into account possible obstructions for the line pieces (queen, rook, bishop). He recalculated the average mobility of these pieces based on various degrees of blockage.

Because a piece can be blocked by a friendly or opposing piece, *mobility* is now determined by counting the number of squares attacked or defended, rather than by simply counting moves as in column 24. For example, a bishop on a1 with a friendly piece on b2 still has *mobility = 1*. That is, it protects b2 and could recapture there if the opponent takes on b2.



The *average mobility of a rook for all degrees of obstruction* (R°) is determined like so:

a) In a corner, mobility ranges from 2 to 14. Two if there are pieces on adjacent squares. Fourteen if the rank and file are open. Average = 8
b) Along the edge of the board (non-corner), the range is 3 to 14. Average = 8.5
c) In the interior of the board, the range is 4 to 14. Average = 9
d) There are 4 corners, 24 edge squares, and 36 interior squares.

$$(4 \times 8) + (24 \times 8.5) + (36 \times 9) = 560$$

$$R° = 560 \div 64 = \mathbf{8.75}$$

Already we have a curious fact. The average mobility of a rook based on possible obstructions is exactly equal to the average mobility of a bishop on an open board!

A similar calculation was done for the *average mobility of a bishop for all degrees of obstruction* (B°). The number of squares attacked by a bishop varies from 1 to 13, depending on its proximity to the centre.

$$(4 \times 4) + (24 \times 4.5) + (20 \times 6.5) + (12 \times 7.5) + (4 \times 8.5) = 378$$

$$B° = 378 \div 64 = \mathbf{5.90625}$$

The value for the queen is the sum of rook and bishop.

$$Q° = 8.75 + 5.90625 = \mathbf{14.65625}$$

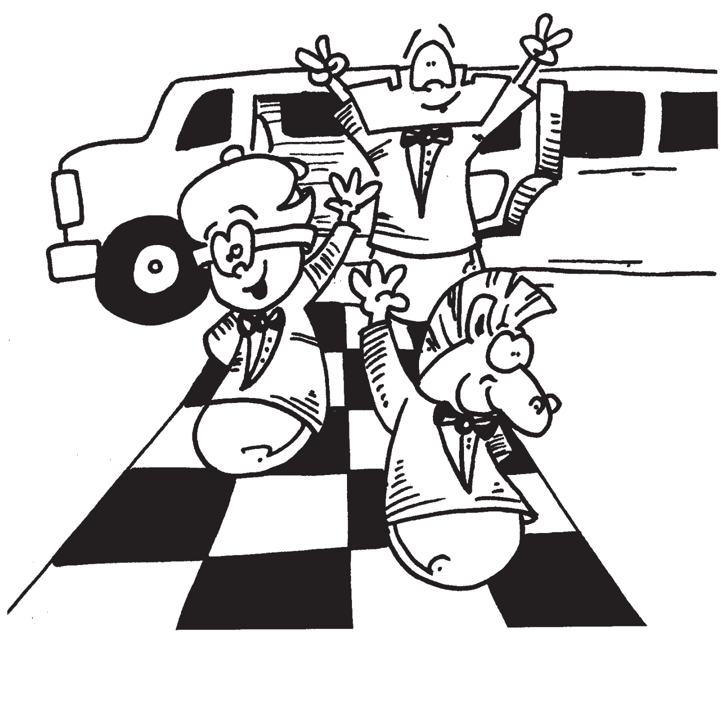
Because the knight cannot be obstructed like a line piece, its value for the recalculation is the same as that on an empty board.

$$N° = \mathbf{5.25}$$

Now we're ready for the next stage of the math adventure. In order to compare the new averages to the accepted standard *value of the pieces* (1-3-3-5-9), François calibrated the mobility numbers so that a knight equals 3, corresponding to its "standard value". To do that, the averages were all multiplied by 3/5.25. Here are the results.

$$N° \times 3/5.25 = 3$$
$$B° \times 3/5.25 = 3.375$$
$$R° \times 3/5.25 = 5$$
$$Q° \times 3/5.25 = 8.375$$

How amazing is that? It's especially neat that the bishop is 3⅜. Many masters and books of instruction give the relative value of a bishop as 3¼ or 3½. Emanuel Lasker judged the queen to be 8½.



Before getting to the problems, there are still the king and pawn to consider. Like the knight, a king cannot be obstructed. Its value in the recalculation is the same as for an empty board.

$$K° = \mathbf{6.5625}$$

$$K° \times 3/5.25 = 3.75$$

The calibrated number fits well with the standard relative values. As a *fighting piece*, Lasker assigned the king 4 points, stronger than a minor piece, weaker than a rook.

A pawn's attack cannot be obstructed. It attacks or defends 1 square when on a side file, and 2 squares if on the other files. The average on the 48 squares where it can stand is easy to figure out.

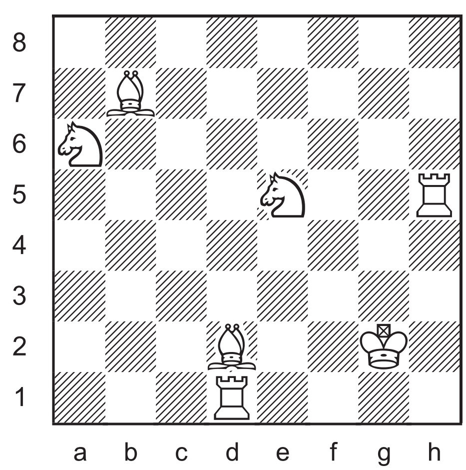
$$(12 \times 1) + (36 \times 2) = 84$$

$$P° = 84 \div 48 = \mathbf{1.75}$$

$$P° \times 3/5.25 = 1$$

That's right, friends. The calibrated value for the pawn is 1. Perfect!

Triple Loyd 72

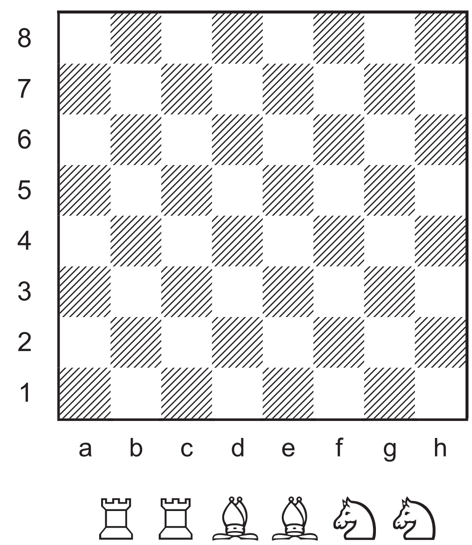


Place the black king on the board so that:

A. Black is in checkmate.
B. Black is in stalemate.
C. White has mate in 1.

All the puzzles in this column feature multiple rooks, bishops, and knights. Let’s count moves again.

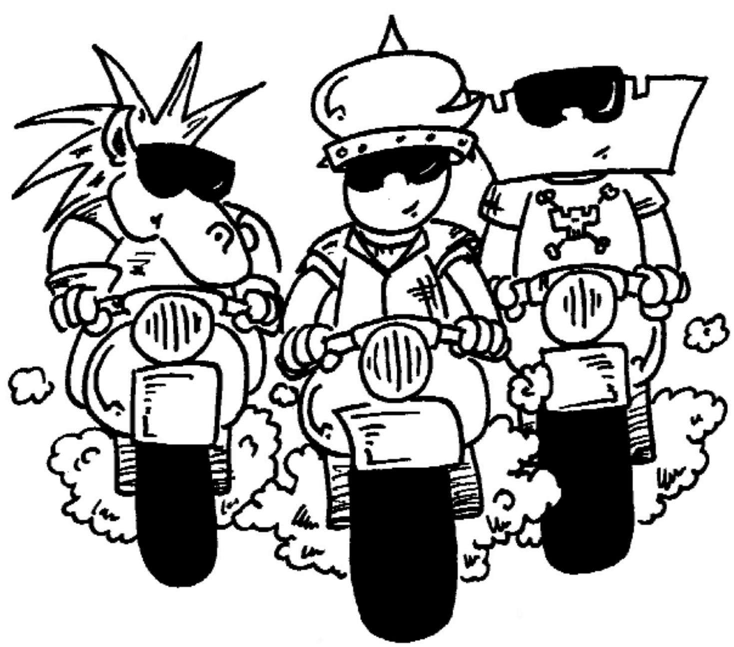
2 Rook 2 Bishop 2 Knight Move Maximizer



Place two rooks, two bishops, and two knights on the board so that they have the most total moves.

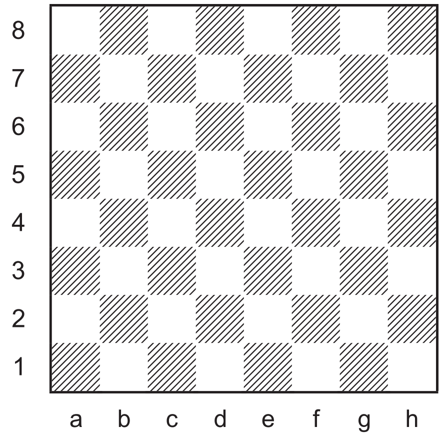
2 Rook 2 Bishop 2 Knight Move Minimizer

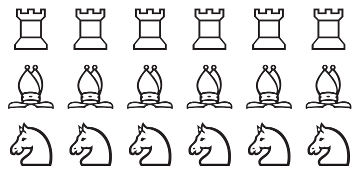
Place two rooks, two bishops, and two knights on the board so that they have the fewest total moves.
The bishops should be on opposite colours.



Mobile Pieces

Rook Bishop Knight Defensive Loops





Place an equal number of rooks, bishops, and knights on the board so that each piece is defended exactly once and each piece defends exactly one other piece.

The chain of defence must form a *continuous loop*. The first piece guards the second; the second guards the third; the third guards the fourth; ...; and the last guards the first.

A. five rooks, five bishops, five knights

B. six rooks, six bishops, six knights

The order of the pieces in the loops does not have to be consistent. For example, with every rook guarding a bishop, and every bishop guarding a knight. It can also be something like R - B - N - B - R - N, etc.

A program by François Labelle, specially written for solving defensive loops, has shown that part B can only be solved with a varied order.

The same problem with four pieces of each type was given in column 24.

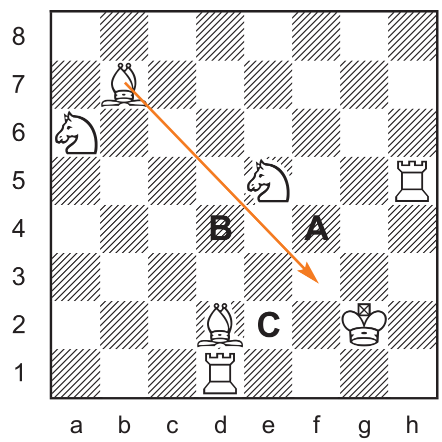
SOLUTIONS

Triple loyd 72 and RRBBNN maximizers/minimizers by J. Coakley. *Puzzling Side of Chess* (2018).

***PDF hyperlinks*.** You can advance to the solution of any puzzle by clicking on the underlined title above the diagram. To return to the puzzle, click on the title above the solution diagram.

***Archives*.** Past columns and a detailed index of problem-types and composers are available in the *Puzzling Side of Chess* archives.

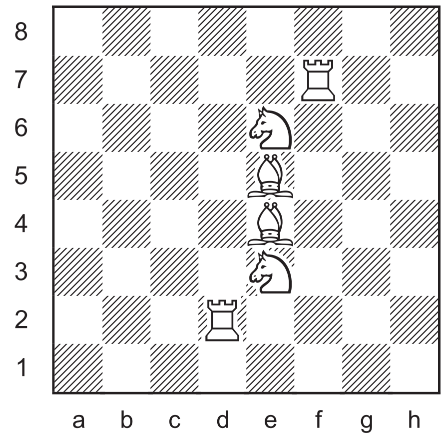
Triple Loyd 72



A. Kf4#
B. Kd4=
C. Ke2 (Bf3#)

As usual, each piece finds a role in some part of the puzzle.

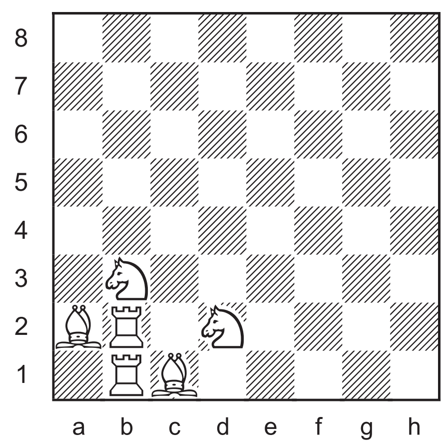
2 Rook 2 Bishop 2 Knight Move Maximizer



70 moves
(R14 + R14 + B13 + B13 + N8 + N8)

This record is the theoretical maximum. There are many solutions.

2 Rook 2 Bishop 2 Knight Move Minimizer



10 moves
(R1 + R1 + B0 + B0 + N4 + N4)

The same position with rook on a1 instead of b2 is also 10 moves.
This record has not been verified by computer.

With same colour bishops, a position with only 9 moves is possible.

Rb1 Rb2 Ba1 Bc1 Nb3 Nd2

(R0 + R2 + B0 + B0 + N3 + N4)

Rook Bishop Knight Defensive Loops

A

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analysed with Caisay 4.1 (Adrian Storisteanu)

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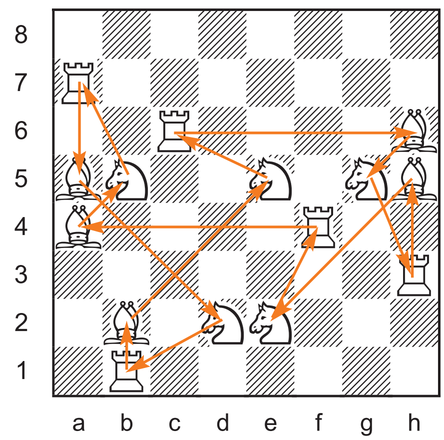
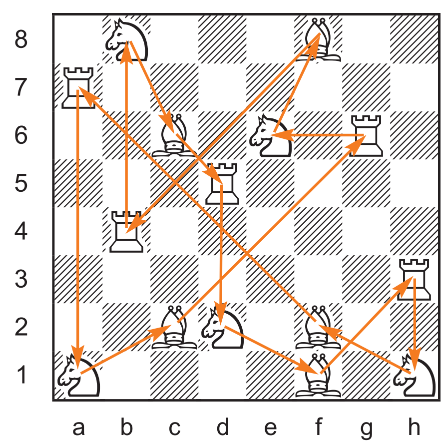
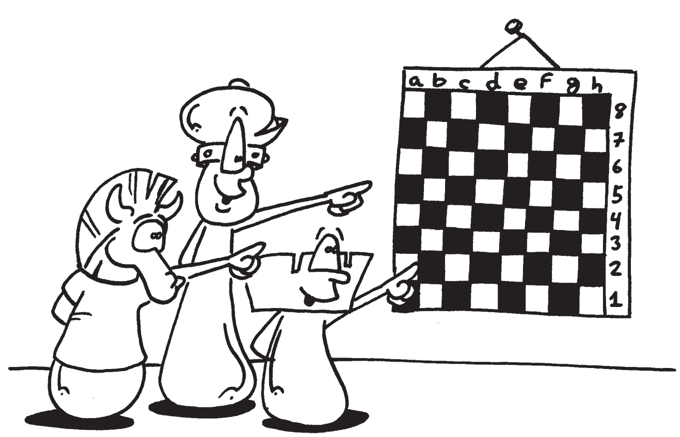


Five rooks, five bishops, five knights. Each defends one piece, each is defended once. There are 25 solutions with a consistent repeating order of R - B - N as in the diagram above, and 2 solutions with the order R - N - B, shown below.



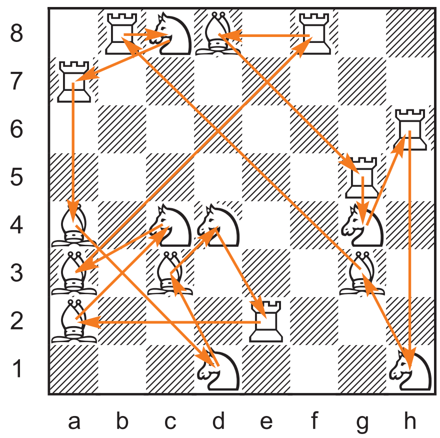


Rook Bishop Knight Defensive Loops

B

François Labelle 2018

Puzzling Side of Chess



Six rooks, six bishops, six knights. Each defends one piece, each is defended once. There are 13 solutions, listed below. The first is shown in the diagram. None of them have a repeating order of pieces.

Merci, François!

Nd1 Bc3 Nd4 Re2 Ba2 Nc4 Ba3 Rf8 Bd8 Rg5 Ng4 Rh6 Nh1 Bg3 Rb8 Nc8 Ra7 Ba4
Re1 Ne4 Rd2 Bh2 Nf4 Bh3 Rc8 Be8 Nb5 Ba3 Nb4 Ra6 Na5 Bb3 Rg8 Nf8 Rh7 Bh4
Rg1 Bg2 Rh3 Ng3 Be2 Nd3 Bb2 Rg7 Ng6 Rf8 Bd8 Ra5 Na6 Bb8 Nd6 Rc4 Nc2 Be3
Rg1 Bg2 Rf3 Bb3 Na4 Bb2 Ng7 Rh5 Bh7 Re4 Nb4 Rc6 Nc7 Ba8 Nb7 Rd8 Nc8 Ba7
Rg1 Ng3 Bf5 Ng6 Rh4 Bh6 Rd2 Ba2 Rf7 Nf6 Re8 Nb8 Ba6 Nb5 Rc3 Bf3 Na8 Bb6
Rh1 Bh2 Nf4 Bh3 Rc8 Ne8 Rd6 Bd4 Nb2 Ba4 Rb5 Nb4 Ba2 Rg8 Nf8 Rh7 Nh4 Bf3
Rh1 Bh2 Rd6 Be6 Rc4 Nb4 Ba2 Nb3 Ra5 Na4 Bb2 Rg7 Ng6 Rf8 Bf3 Ng2 Be3 Nf2
Rh1 Bh2 Nc7 Ra6 Na3 Bc2 Nd3 Rb4 Bd4 Nc3 Ba2 Rf7 Ne7 Rg8 Bg4 Rh5 Nh4 Bg2
Rh1 Bh2 Rb8 Bc8 Rd7 Ng7 Bh5 Ne2 Rc3 Nc4 Ra5 Ng5 Bh7 Ng8 Rf6 Bf2 Nh4 Bg2
Rh1 Bh2 Rb8 Bc8 Nf5 Re3 Ne8 Bf6 Ng5 Rh7 Bh5 Ng6 Bf8 Nc5 Ra4 Ba5 Rd2 Nf2
Rh1 Bh2 Rb8 Nc8 Ra7 Ba5 Nc3 Ba2 Re6 Nf6 Rg8 Bf8 Na3 Bc2 Nf5 Rd4 Nd2 Bf3
Rh1 Nh3 Rf4 Bf6 Nh8 Bg6 Re8 Bf8 Nh6 Bg8 Nb3 Ra5 Ba7 Nb6 Rd7 Bb7 Rg2 Ng3
Rh1 Bh4 Nf6 Rg8 Bf8 Nc5 Rb3 Bc3 Rd4 Nf4 Re2 Ne3 Bf5 Nc8 Ra7 Bc7 Ne5 Bc6

Until next time.