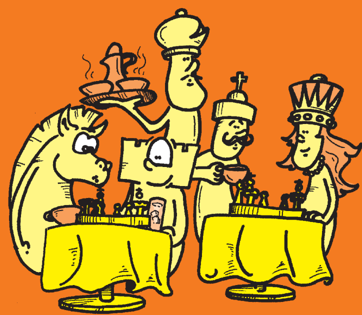


THE PUZZLING SIDE OF CHESS

Jeff Coakley

BOARD DOMINATION

number 6 July 28, 2012

This edition of *The Puzzling Side of Chess* features “board domination” puzzles. The task is to place pieces on the chessboard so that all squares are attacked or occupied.

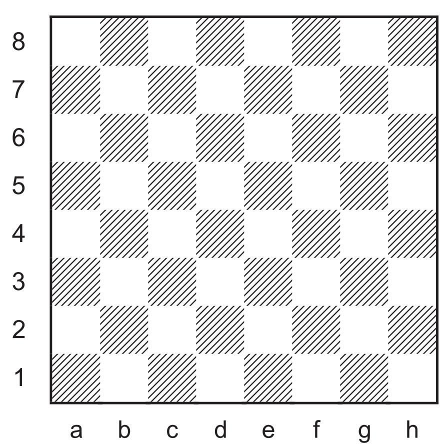
There are three types of board domination.

1. Every square is either attacked or occupied. This is the general definition of *domination*.

2. Every empty square is attacked, and none of the pieces attack each other. In other words, the occupied squares are not attacked. This special case is known as *independent domination.*

3. Every empty square is attacked, and each of the pieces is attacked. This kind of arrangement, with all occupied squares attacked, is called *total domination.*

The following position is very interesting. It is a common starting place for many great puzzles.

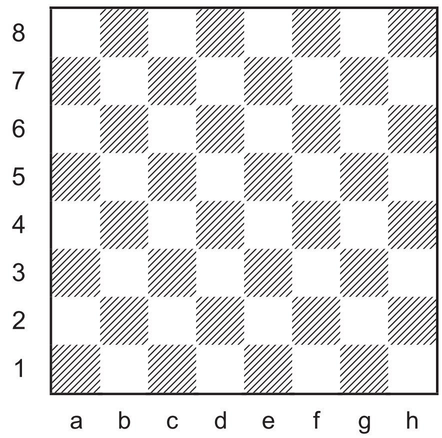


An empty board! As most chess players realize, the possibilities are endless.

Five Queens

"Five queens" is one of the oldest and most famous domination puzzles. Russian master Carl Jaenisch included it in his 1863 book *Traité des applications de l'analyse mathématique au jeu des échecs*.

Five Queens



A. Place five queens on the board so that every square is either attacked or occupied.

B. Place five queens on the board so that every empty square is attacked and none of the queens attack each other.

C. Place five queens on the board so that every empty square is attacked and every queen is attacked.

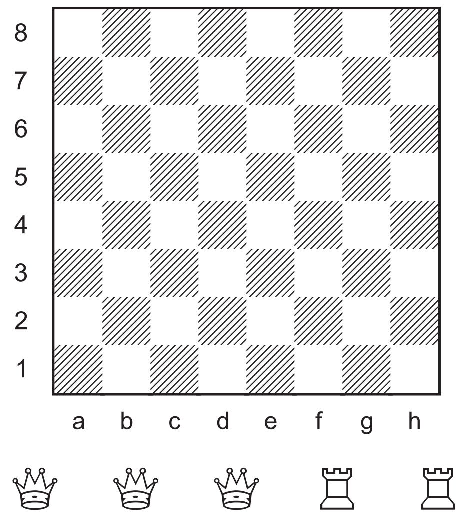
Mathematicians have done a lot of work on the five queen puzzle because of its relationship to more complicated non-chess situations. Formulas have been developed to determine the number of "pieces" necessary to dominate "grids" of any size.

Five queens is also an awesome poker hand. If you have a wild card!?

Three Queens & Two Rooks

This puzzle dates back more than one hundred years. Dr. Charles Planck, a British surgeon who wrote several magazine articles on *magic squares* and other puzzles, discovered the unique solution to part B.

<u>Three Queens & Two Rooks</u>



A. Place three queens and two rooks on the board so that every empty square is attacked and none of the pieces attack each other. (independent domination)

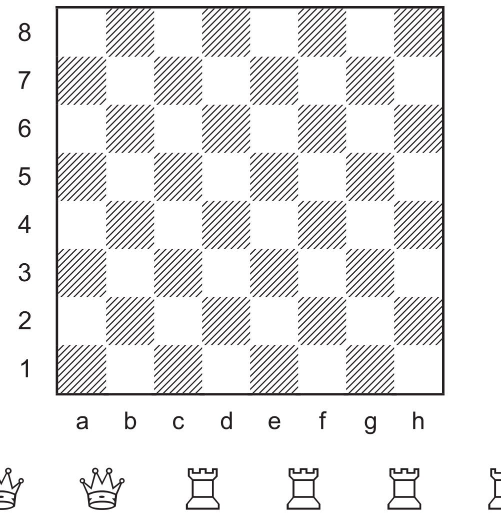
B. Place three queens and two rooks on the board so that every empty square is attacked and every piece is attacked. (total domination)

My deepest thanks to Noam Elkies, Professor of Mathematics at Harvard University, who provided a symmetrical solution for total domination by five queens, as well as a solution for independent domination by three queens and two rooks. He also “computed” the number of solutions for total domination by five queens and for both types of domination with three queens and two rooks.

Most of you would probably agree that the concept of board domination makes for a fun puzzle. And like any grand idea, we can grind it into the ground!

For those people who can’t get enough of a good thing, here are two more puzzles. I doubt they are original, but I haven’t seen them anywhere before.

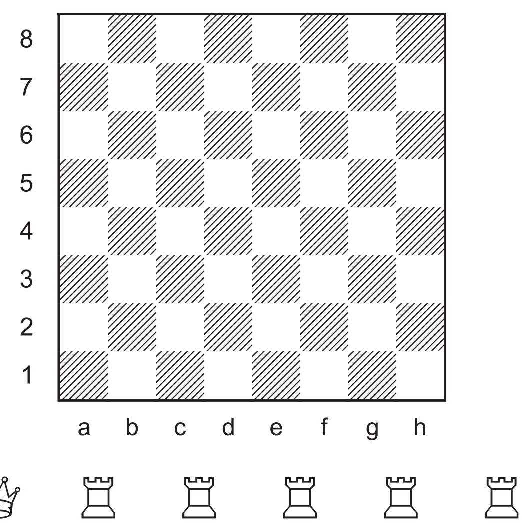
Two Queens & Four Rooks



A. Place two queens and four rooks on the board so that every empty square is attacked and none of the pieces attack each other.

B. Place two queens and four rooks on the board so that every empty square is attacked and every piece is attacked.

One Queen & Five Rooks



A. Place one queen and five rooks on the board so that every empty square is attacked and none of the pieces attack each other.

B. I believe a total domination solution is impossible, but please try if you like!

Board domination is not restricted to queens and rooks. In future columns, we will look at puzzles with dominating knights and other combinations of pieces.

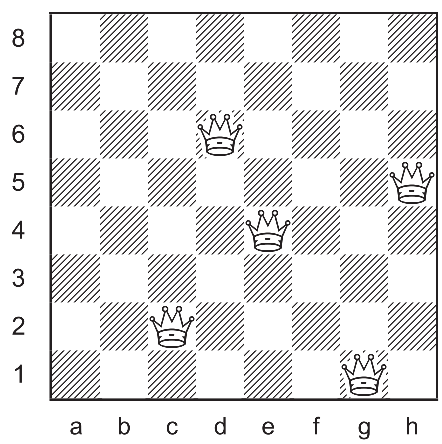
SOLUTIONS

PDF hyperlinks. You can advance to the solution of any puzzle by clicking on the underlined title above the diagram. To return to the puzzle, click on the title above the solution diagram.

Five Queens

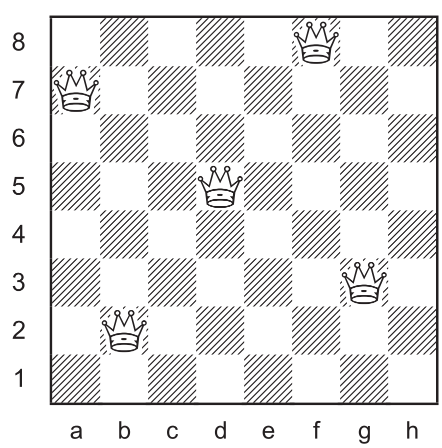
Did you know there are 7.6 million ways to arrange five queens on a chessboard!?

A. There are **4860** solutions for general domination by five queens. Here is one example where some queens are attacked and some are not.



B. There are **91** distinct solutions for independent domination by five queens. This number was established by Carl Jaenisch in 1863. I wonder how he did it without a computer!

All of the solutions are asymmetrical, so each of them can be rotated and reflected, giving **728** as the total number of solutions (91 x 8). The arrangement shown in this diagram has an impressive geometry.

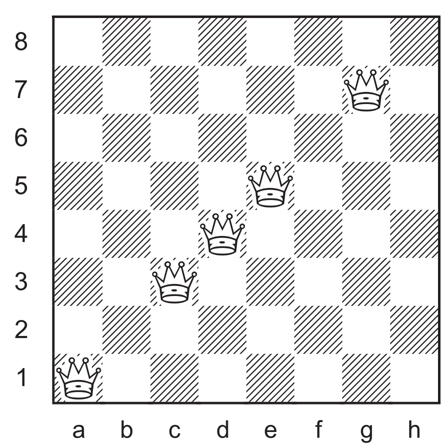


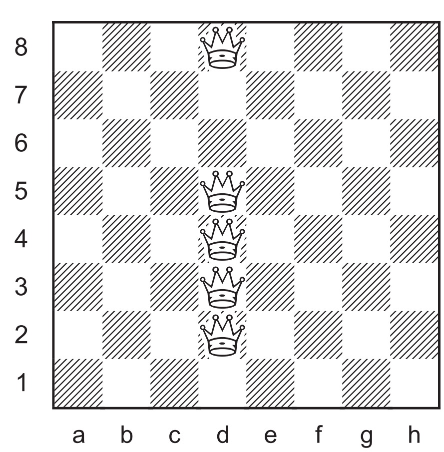
Some other solutions for independent domination:

Qc6 Qd3 Qe5 Qf7 Qg4
Qa3 Qc7 Qd4 Qe1 Qg5
Qa2 Qc8 Qd4 Qg5 Qh1
Qa2 Qb7 Qd1 Qe3 Qf6

C. There are **56** distinct solutions for total domination by five queens, including 24 that are symmetrical about a long diagonal. So after rotation and reflection, the total number of solutions is **352**. (24 x 4) + (32 x 8)

The two examples below, with all the queens on a single line, are from *Amusements in Mathematics, a* 1917 book by noted English puzzlist Henry Dudeney.

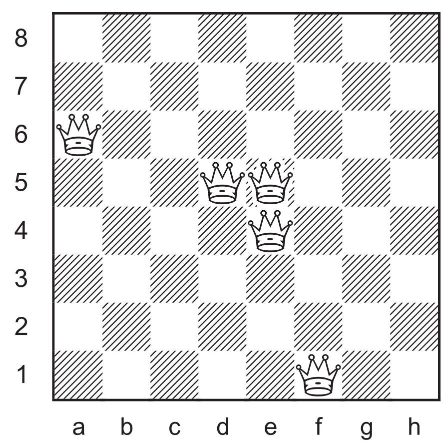




Some other solutions for total domination:

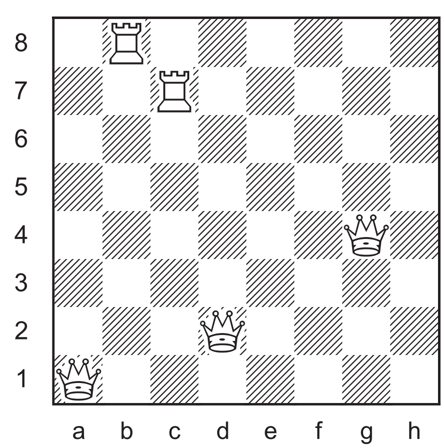
Qa1 Qc3 Qe5 Qf6 Qg7 (long diagonal, different spacing)
Qb1 Qd3 Qe4 Qf5 Qh7 (non-long diagonal)
Qb2 Qc4 Qd5 Qe6 Qg7 (two diagonals)
Qa2 Qb1 Qc8 Qe6 Qg4 (two diagonals)
Qa4 Qc6 Qe4 Qe8 Qg2 (cross diagonals)
Qa5 Qd5 Qe5 Qf5 Qh1 (four queens on same rank)
Qc4 Qd6 Qe3 Qe4 Qe5 (non-linear)
Qa5 Qc1 Qe1 Qg3 Qg7 (non-linear, no adjacent queens)

The following symmetrical solution deserves a diagram.

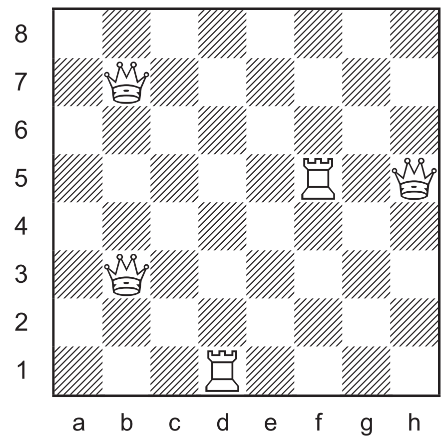


<u>Three Queens & Two Rooks</u>

A. There are **16** distinct solutions for independent domination. None of them are symmetrical, so with rotation and reflection, there are a total of **128** solutions. This diagram shows one of them.



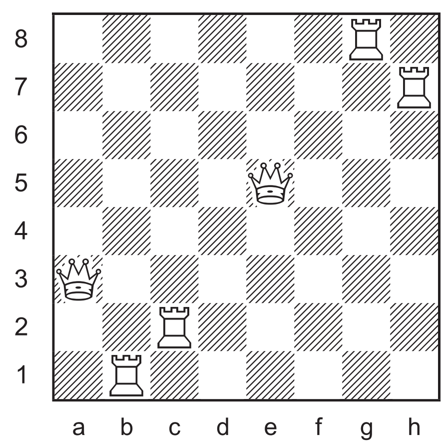
B. The solution for total domination is unique. The diagrammed position (by Dr. Planck) was mentioned by Alain White in his 1913 book *Sam Loyd and His Chess Problems*. It can be rotated and reflected to give eight different "views" of the same arrangement.



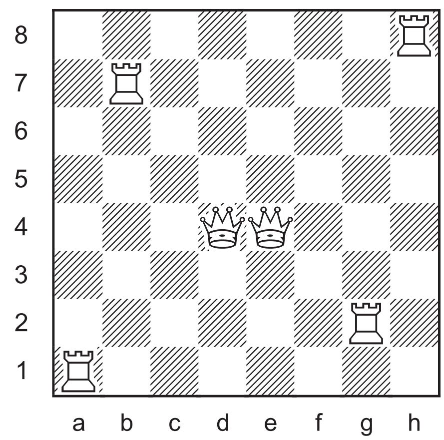
<u>**Two Queens & Four Rooks**</u>

This should be a breeze after the previous puzzles.

A. There are many solutions for independent domination.

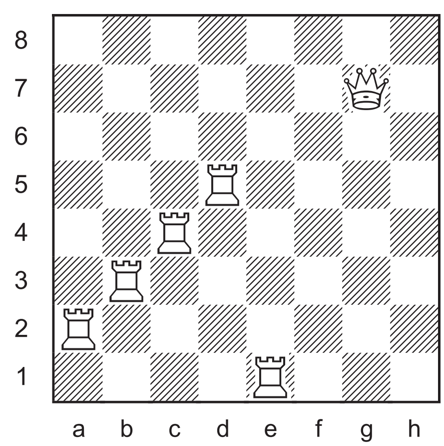


B. There are many solutions for total domination.



One Queen & Five Rooks

A. There are many solutions for independent domination.



B. Total domination. Did you give up yet?

[*Since this column was first published, computer analysis has shown that total domination with Q+ 5R is impossible.*]

Until next time!